MENIIT

NEET | IIT-JEE | FOUNDATION

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JEE MAIN-2020

COMPUTER BASED TEST (CBT)

DATE : 04-09-2020 (SHIFT-1) | TIME : (9.00 am to 12.00 pm)

Duration 3 Hours | Max. Marks : 300

QUESTION & SOLUTIONS

PART-A : PHYSICS

SECTION – 1 : (Maximum Marks : 80)

Single Choice Type

This section contains 20 Single choice questions. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which Only One is correct.

Full Marks : +4 If ONLY the correct option is chosen.

Negative Marks : –1 (minus one) mark will be deducted for indicating incorrect response.

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1. A small bar magnet is placed with its axis at 30° with an external magnetic field of 0.06 T experiences a torque of 0.018 Nm. The minimum work required to rotate it from its stable to unstable equilibrium position is:

(1) 9.2×10^{-3} J (2) 11.7×10^{-3} J (3) 6.4×10^{-2} J (4) 7.2×10^{-2} J

Ans. (4)

Sol. $\tau = MB\sin\theta = 0.18$

$$M = \frac{0.018}{B\sin\theta} = \frac{0.018}{0.06 \times 0.5} = 0.6A - m^2$$

$\omega = \Delta U = U_f - U_i$

$= -MB\cos 180° - (-MB\cos 0°)$

$= 2MB$

$= 2 \times 0.6 \times 0.06$

$= 0.072$ J

2. On the x-axis and at a distance x from the origin, the gravitational field due to a mass distribution is given by $\frac{Ax}{(x^2+a^2)^{3/2}}$ in the x-direction. The magnitude of the gravitational potential on the x-axis at a distance x, taking its value to be zero at infinity is:

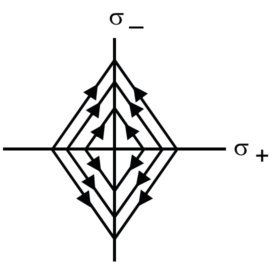
(1) $\frac{A}{(x^2+a^2)^{3/2}}$ (2) $A(x^2+a^2)^{3/2}$ (3) $A(x^2+a^2)^{1/2}$ (4) $\frac{A}{(x^2+a^2)^{1/2}}$

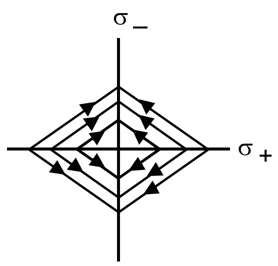
Ans. (4)

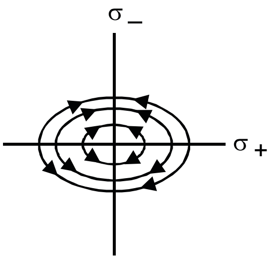
Sol. $$V_x = \int_{\infty}^{x} \frac{Ax}{(x^2+a^2)^{3/2}}(-dx)$$

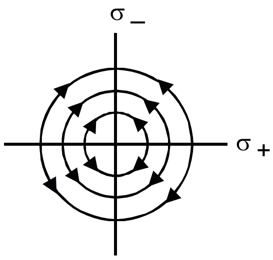
$$V_x = -\frac{A}{\sqrt{A^2+x^2}}$$

3. Two charged thin infinite plane sheets of uniform charge density σ_+ and σ_-, where $|\sigma_+| > |\sigma_-|$, intersect at the right angle. Which of the following best represents the electric field lines for the system:

(1) 

(2) 

(3) 

(4) 

Ans. (2)

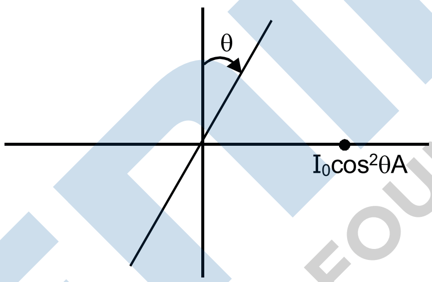
Sol. The electric field intensity due to each uniformly charged infinite plane is uniform. The electric field intensity at points A, B, C and D due to plane 1, plane 2 and both planes are given by E_1, E_2 and E as shown in figure 1. Hence the electric lines of forces are as given in figure 2.

4. A beam of plane polarised light of large cross-sectional area and uniform intensity of 3.3 Wm^{-2} falls normally on a polariser(cross sectional area 3×10^{-4} m^2) which rotates about its axis with an angular speed of 31.4 rad/s. The energy of light passing through the polariser per revolution, is close to:

(1) 1.0×10^{-4} J (2) 5.0×10^{-4} J (3) 1.0×10^{-5} J (4) 1.5×10^{-4} J

Ans. (1)

Sol.



Average energy = $I_0A < \cos^2\underline{\theta} >$

$$= \frac{3.3 \times 3 \times 10^{-4}}{2} = \frac{9.9}{2} \times 10^{-4} = 4.95 \times 10^{-4}$$

5. Choose the correct option relating wavelengths of different parts of electromagnetic wave spectrum:

(1) $\lambda_{\text{radio waves}} > \lambda_{\text{micro waves}} > \lambda_{\text{visible}} > \lambda_{\text{x-rays}}$

(2) $\lambda_{\text{visible}} < \lambda_{\text{micro waves}} < \lambda_{\text{radio waves}} < \lambda_{\text{x-rays}}$

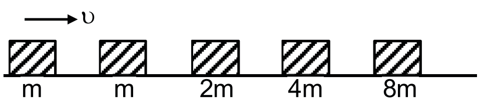
(3) $\lambda_{\text{visible}} > \lambda_{\text{x-rays}} > \lambda_{\text{radio waves}} > \lambda_{\text{micro waves}}$

(4) $\lambda_{\text{x-rays}} < \lambda_{\text{micro waves}} < \lambda_{\text{radio waves}} < \lambda_{\text{visible}}$

Ans. (1)

Sol. Theory based

6. Blocks of masses m, 2m, 4m and 8m are arranged in a line of a frictionless floor. Another block of mass m, moving with speed υ along the same line (see figure) collides with mass m in perfectly inelastic manner. All the subsequent collisions are also perfectly inelastic. Bt the time the last block of mass 8m starts moving the total energy loss is p% of the original energy. Value of 'p' is close to :



(1) 87 (2) 37 (3) 77 (4) 94

Ans. (4)

m → v m 2m 4m 8m **Sol.**

Inelastic collision

$mv = 16\, mv^1$

$v^1 = \frac{v}{16}$

$\Delta K \text{ loss} = \frac{1}{2}mv^2 - \frac{1}{2}(16M)\left(\frac{v}{16}\right)^2$

$= \frac{1}{2}mv^2 - \frac{1}{2}M\frac{v^2}{16}$

$= \frac{1}{2}mv^2\left(\frac{15}{16}\right)$

$\%\Delta K \text{ loss} = \frac{\frac{1}{2}mv^2\left(\frac{15}{16}\right)}{\frac{1}{2}MV^2} \times 100 = \frac{15}{16} \times 100 = 93.75\%$

7. A particle of mass $m_A = m/2$ moving along the x-axis with velocity v_0 collides elastically with another particle B at rest having mass $m_B = m/e$. If both the particles move along the x-axis after the collision, the change $\Delta\lambda$ in the wavelength of the particle A, in terms of its de-Broglie wavelength (λ_0) before the collision is :

(1) $\Delta\lambda = 2\lambda_0$ (2) $\Delta\lambda = 4\lambda_0$ (3) $\Delta\lambda = \frac{3}{2}\lambda_0$ (4) $\Delta\lambda = \frac{5}{2}\lambda_0$

Ans. (2)

Sol. $v_1 = \frac{2(m/3)0}{\left(\frac{m}{2}+\frac{m}{3}\right)} + \frac{\left(\frac{m}{2}-\frac{m}{3}\right)v}{\left(\frac{m}{2}+\frac{m}{3}\right)} = \frac{v}{5}$

m/2 → v m/3 (rest) = m/2 → V_1 m/3 → V_2

For particle A,

Initial de-Broglie wavelength

$\lambda_0 = \frac{h}{\frac{m}{2}v} = \frac{2h}{mv}$

Final de-Broglie wavelength after collision.

$\lambda_1 = \frac{h}{\frac{m}{2}\frac{v}{5}} = \frac{10h}{mv} = 5\lambda_0$

Change in De-Broglie wavelength $\Delta\lambda = \lambda_1 - \lambda_0 = 4\lambda_0$

8. A battery of 3.0 V is connected to a resistor dissipating 0.5 W of power. If the terminal voltage of the battery is 2.5 V, the power dissipated within the internal resistance is :

(1) 0.125 W (2) 0.50 W (3) 0.10 W (4) 0.072 W

Ans. (3)

Sol. E = 3V

$V_R = 2.5V$

By KVL

$V_r + V_R = E$

$V_r + 2.5 = 3$

$V_r = 0.5$

$$\frac{V_R}{V_r} = \frac{IR}{Ir} = \frac{2.5}{0.5} = 5 \qquad ......(1)$$

$$\frac{R}{r} = 5$$

$$\frac{P_R}{P_r} = \frac{I^2R}{I^2r} = \frac{R}{r}$$

$$\frac{P_R}{P_r} = 5$$

$$P_r = \frac{P_R}{5} = \frac{0.5}{5} = 0.1 \text{ watt}$$

9. The specific heat of water = 4200 J-kg^{-1} K^{-1} and the latent heat of ice = 3.4 × 10^5 J-kg^{-1}.100 grams of ice at 0ºC is placed in 200 g of water at 25ºC. The amount of ice that will melt as the temperature of water reaches 0ºc is close to (in grams)

(1) 61.7 (2) 69.3 (3) 64.6 (4) 63.8

Ans. (1)

Sol. $MS\Delta T = M\ell$

$$\frac{200}{1000} \times 4200 \times 25 = m \times 340 \times 10^3$$

m = 61.7

10. Match the C_p/C_v ratio for ideal gases with different type of molecules:

Molecule Type	C_p / C_v
(A) Monoatomic	(I) 7/5
(B) Diatomic rigid molecules	(II) 9/7
(C) Diatomic non-rigid molecules	(III) 4/3
(D) Triatomic rigid molecules	(IV) 5/3

(1) (A) – (III), (B) – (IV), (C) – (II), (D) – (I)
(2) (A) – (IV), (B) – (I), (C) – (II), (D) – (III)
(3) (A) – (IV), (B) – (II), (C) – (I), (D) – (III)
(4) (A) – (II), (B) – (III), (C) – (I), (D) – (IV)

Ans. (2)

Sol. $\gamma = 1 + 2/f$

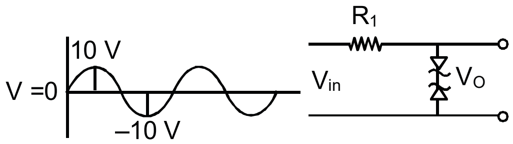
For A ; f = 3 ; $\gamma = 1 + \frac{2}{3} = \frac{5}{3}$

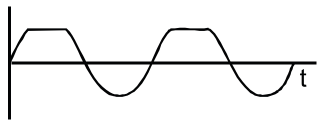
For B ; f = 5 ; $\gamma = 1 + \frac{2}{5} = \frac{7}{5}$

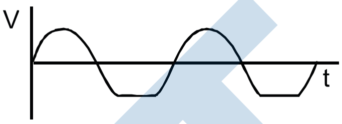
For C ; f = 7 ; $\gamma = 1 + \frac{2}{7} = \frac{9}{7}$

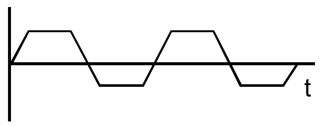
For D ; f = 6 ; $\gamma = 1 + \frac{2}{6} = \frac{4}{3}$

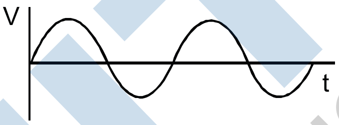
11. Take the breakdown voltage of the zener diode used in the given circuit as 6V. For the input voltage shown in the figure below, the time variation of the output voltage is : (Graphs drawn are schematic and not to the scale)



(1) 

(2) 

(3) 

(4) 

Ans. (3)

Sol. Based on Theory

12. A air bubble of radius 1 cm in water has an upward acceleration of 9.8 cms^{-2}. The density of water is 1 gm cm^{-3} and water offers negligible drag force on the bubble. The mass of the bubble is (g = 980 cm/s^2).

(1) 4.15 gm (2) 1.52 gm (3) 4.51 gm (4) 3.15 gm

Ans. (1)

Sol. $F_b - mg = ma$

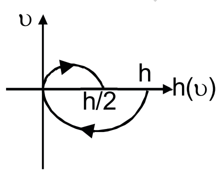
$F_b = m(g + a)$

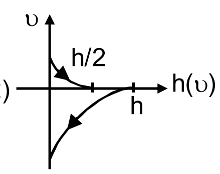
$$= \frac{4}{3}\pi r^3 \rho g = m(g + a)$$

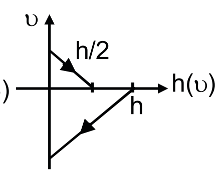
$$m = \frac{4}{3}\pi(1)g = m(980 + 9.8)$$

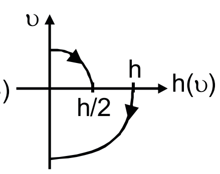
m = 4.15

13. A tennis ball is released from a height h and after freely on a wooden floor it rebounds and reaches height h/2. The velocity versus height of the ball during its motion may be represented graphically by : (graphs are drawn schematically and on not to scale)

(1) 

(2) 

(3) 

(4) 

Ans. (4)

Sol. (i) For uniformly accelerated / deaccelerated motion

$v^2 = u^2 \pm 2gh$

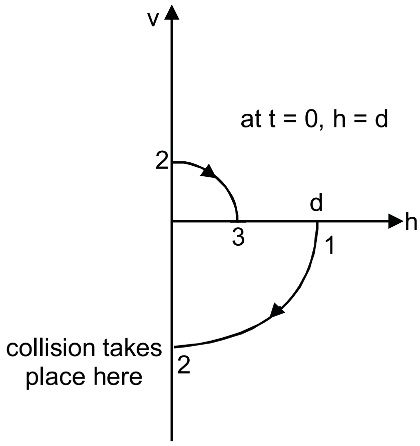
i.e. v-h graph will be a parabola (because equation is quadratic).

(ii) Initially velocity is downwards (-ve) and then after collision it reverses its direction with lesser magnitude. i.e. velocity is upwards (+ve). Graph (A) satisfies both these conditions.

Therefore, correct answer is (A)

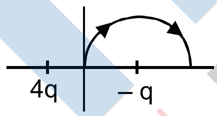
Note that time t = 0 corresponds to the point on the graph where h = d

Next time collision takes place at 3.



$1 \rightarrow 2$: V increases downwards
At $2 \rightarrow$ velocity changes its direction
$2 \rightarrow 3$ V decreases upwards

14. A two point charges 4q and –q are fixed on the x-axis at x = -d/2 and x= d/2, respectively. If the third point charge 'q' is taken from the origin to x= d along the semicircle as shown in the figure, the energy of the charge will :



(1) Increase by $\frac{2q^2}{3\pi \epsilon_0 d}$

(2) Increase by $\frac{2q^2}{4\pi \epsilon_0 d}$

(3) decrease by $\frac{q^2}{4\pi \epsilon_0 d}$

(4) decrease by $\frac{4q^2}{3\pi \epsilon_0 d}$

Ans. (4)

Sol. Potential at O,

$$\Rightarrow \quad V_O = \frac{K4q}{\frac{d}{2}} + \frac{K(-q)}{\frac{d}{2}} = \frac{6Kq}{d}$$

Potential at P,

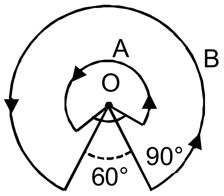
$$\Rightarrow \quad V_P = \frac{K4q}{\frac{3d}{2}} + \frac{K(-q)}{\frac{d}{2}} = \frac{2Kq}{3d}$$

Change in potential energy of a charge q = $q\Delta V = q(V_f - V_i)$

$= q(V_P - V_O)$

$$q\left(\frac{2Kq}{3d} - \frac{6Kq}{d}\right) = \frac{16q^2}{4\pi\varepsilon_0 3d} = \frac{4q^2}{3\pi\varepsilon_0 d}$$

15. A wire A, bent in the shape of an arc of a circle, carrying a current of 2A and having radius 2 cm and another wire B, also bent in the shape of an arc of a circle, carrying a current of 3A and having radius of 4 cm, are placed as shown in the figure. The ratio of the magnetic fields due to the wires A and B at the common centre O is :



(1) 6 : 5 (2) 6 : 4 (3) 2 : 5 (4) 4 : 6

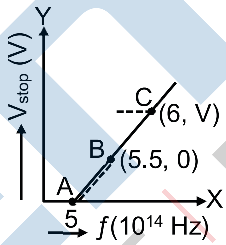
Ans. (1)

Sol. $B_C = \frac{\mu_0 I}{4\pi R}(\theta)$ (θ angle substance at centre)

$$\frac{B_{large}}{B_{small}} = \frac{i_1}{i_2} \times \frac{R_2(2\pi - \pi/2)}{R_1(2\pi - \pi/3)}$$

$$= \frac{2}{2} \times \frac{2}{3} \times \frac{3\pi}{2} \times \frac{3}{5\pi} = \frac{6}{5}$$

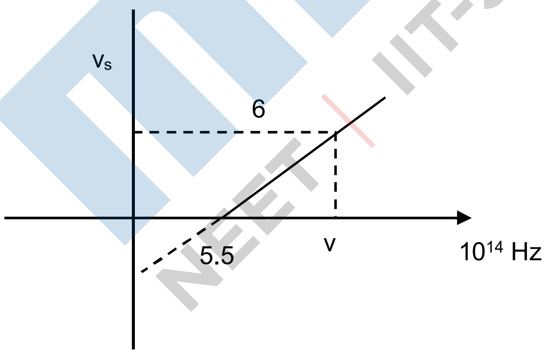
16. Given figure shows few data points in a photo-electric effect experiment for a certain metal. The minimum energy for ejection of electrons from its surface is : (Planck's constant h = 6.62 × 10–34 J-s)



(1) 2.10 eV (2) 2.59 eV (3) 1.93 eV (4) 2.27 eV

Ans. (4)

Sol.



Threshold Energy = hv

= 6.62 × 10^{-34} × 5.5 × 10^{14} J

Work friction $= \frac{6.62 \times 5.5 \times 10^{-20}}{1.6 \times 10^{-19}} eV = 2.27\,eV$

17. For a transvers wave travelling, along a straight line, the distance between two peaks (crests) is 5m, while the distance between one crest and one trough is 1 : 5 m. The possible wavelengths (in m) of the waves are :

(1) 1, 3, 5 (2) $\frac{1}{2},\frac{1}{4},\frac{1}{6},.........$ (3) $\frac{1}{1},\frac{1}{3},\frac{1}{5},.........$ (4) 1,2,3,……..

Ans. (3)

Sol. Trough to crest distance

$1.5 = (2n_1 + 1)\frac{\lambda}{2}$ …(1)

Trough to trough distance

$5 = (n_2\lambda)$ …(2)

from (1) and (2)

$\frac{1.5}{5} = \frac{2n_1 + 1}{2(n_2)}$

$3n_2 = 10n_1 + 5$

n_1 and n_2 are integer

(1) $n_1 = 1$, $n_2 = 5$, $\lambda = 1$

(2) $n_1 = 4$, $n_2 = 15$, $\lambda = 1/3$

(3) $n_1 = 7$. $n_2 = 25$, $\lambda = 1/5$

18. Standing from the origin at time t = 0, with initial velocity $5\hat{j}ms^{-1}$, a particle moves in the x-y plane with a constant acceleration of $(10\hat{i} + 4\hat{j})ms^{-2}$. At time t, its coordinates are (20m, y_0 m). The values of t and y_0 are, respectively :

(1) 4s and 52 m (2) 2s and 24 m (3) 2s and 18 m (4) 5s and 25 m

Ans. (3)

Sol. Equation (1)

$S_x = \frac{1}{2}a_x t^2$

$20 = \frac{1}{2} \times 10 \times t^2$

t = 2

Equation (2)

$S_y = u_y t + \frac{1}{2}a_y t^2$

$y = 5(2) + \frac{1}{2}(4)(2)^2$

y = 18

19. Dimensional formula for thermal conductivity is (here K denotes the temperature) :

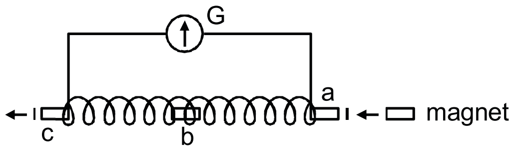
(1) MLT^{-2} K (2) MLT^{-3} K^{-1} (3) MLT^{-3} K (4) MLT^{-2} K^{-2}

Ans. (2)

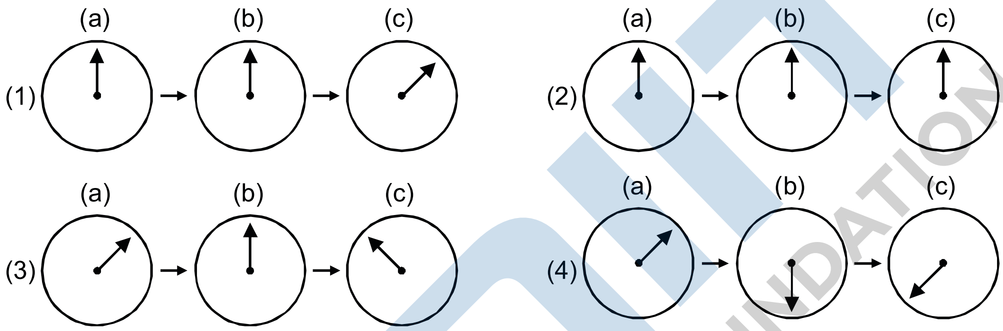
Sol. $k = \dfrac{(Q/t)\Delta x}{A\Delta T}$

$= \dfrac{M^1L^2T^{-2}(L)}{L^2K(T)} = M^1L^1T^{-3}K^{-1}$

20. A small bar magnet is moves through a coil at constant speed from one end to the other. Which of the following series of observations will be seen on the galvanometer G attached across the coil ?



Three positions shown describe : (a) the magnet's entry (b) magnet is completely inside and (c) magnet's exit.



Ans. (3)

Sol. Theory Based

SECTION – 2 : (Maximum Marks : 20)

This section contains FIVE (05) questions. The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto one digit.

If the numerical value has more than two decimal places truncate/round-off the value upto TWO decimal places.

Full Marks : +4 If ONLY the correct option is chosen.

Zero Marks : 0 In all other cases

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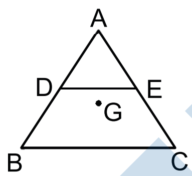
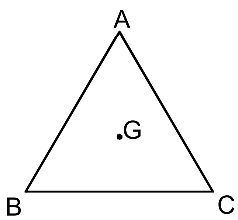
21. ABC is a plane lamina of the shape of an equilateral triangle. D, E are mid-points of AB, AC and G is the centroid of the lamina. Moment of inertia of the lamina about a axis passing through G and perpendicular to the plane ABC is I_0. If part ADE is removed, the moment of inertia of the remaining part about the same axis is $\frac{NI_0}{16}$ where N is an integer. Value of N is :



Ans. 11

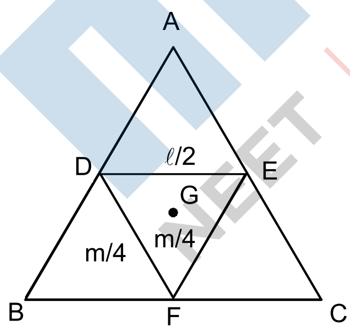
Sol.



Let mass of lamina = m, and length of side = ℓ, then moment of inertia of lamina about an axis passing through G and perpendicular to the plane.

$I_0 \propto m\ell^2$

$I_0 = km\ell^2$



Let moment of inertia of DEF = I_1 about G

then, $I_1 \propto \left(\frac{m}{4}\right)\left(\frac{\ell}{2}\right)^2 \propto \frac{m\ell^2}{16}$

$I_1 = \frac{I_0}{16}$

Let $I_{ADE} = I_{BDF} = I_{EFC} = I_2$

then, $3I_2 + I_1 = I_0$

$3I_2 + \frac{I_0}{16} = I_0 \quad \Rightarrow I_2 = \frac{5I_0}{16}$

So, Moment of inertia of DECB = $2I_2 + I_1 = 2\left(\frac{5I_0}{16}\right) + \left(\frac{I_0}{16}\right) = \frac{11I_0}{16}$

22. In a compound microscope, the magnified virtual image is formed at a distance of 25 cm from the eyepiece. The focal length of its objective lens is 1cm. If the magnification is 100 and the tube length of the microscope is 20 cm, then the focal length of the eye-piece lens (in cm) is ________

Ans. NTA answer is 5 and Reso answer is 6.25

Sol. $M = \frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right)$

$M = \frac{L}{f_0}\left(1 + \frac{D}{f_e}\right)$

$100 = \frac{20}{(1)}\left(1 + \frac{25}{f_e}\right); 5 = 1 + \frac{25}{f_e}$

$4 = \frac{25}{f_e}; f_e = \frac{25}{4} = 6.25 \text{ cm}$

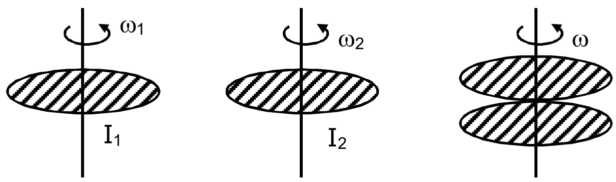
{It is not mentioned that the answer should be an integer so better answer should be 6.25}

23. A circular disc of mass M and radius R is rotating about its axis with angular speed ω_1. If another stationary disc having radius $\frac{R}{2}$ and same mass M is dropped co-axially on to the rotating disc. Gradually both discs attain constant angular speed ω_2. The energy lost in the process is p% of the initial energy. Value of p is ___________

Ans. 20

Sol. Angular momentum conservation

$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2) \times \omega_f$



$\frac{MR^2}{2} \times \omega + 0 = \left(\frac{MR^2}{2} + \frac{MR^2}{8}\right)\omega_r$

$\omega_r = \frac{4}{5}\omega$

Final K.E., $K_f = \frac{1}{2}\left(\frac{MR^2}{2} + \frac{MR^2}{8}\right)\frac{16}{25}\omega^2$

$K_f = \frac{MR^2\omega^2}{5}; K_i = \frac{1}{2}\left(\frac{MR^2}{2}\right)\omega^2 = \frac{MR^2\omega^2}{4}$

Percentage loss in kinetic energy % loss $= \dfrac{\dfrac{MR^2\omega^2}{4} - \dfrac{MR^2\omega^2}{5}}{\dfrac{MR^2\omega^2}{4}} \times 100 = 20\%$

24. A closed vessel contains 0.1 mole of a monoatomic ideal gas at 200 K. If 0.05 mole of the same gas at 400 K is added to it, the final equilibrium temperature (in K) of the gas in the vessel will be close to_______

Ans. NTA answer is 266 and Reso answer is 266.67 $\approx$ 267

Sol. (0.1) (200) + (0.05) (400) = (0.15)T

$$T = \frac{20+20}{0.15} = \frac{800}{3} = 266.67$$

= 266.67 {It is not mentioned that answer should be in integer, so 266.67 will be better answer}

25. In the line spectra of hydrogen atom, difference between the largest and the shortest wavelengths of the Lyman series is 305 Å. The corresponding difference for the Paschan series in Å is :________

Ans. NTA answer is 10553 and Reso answer is 10553.14

Sol. Lyman ; $\dfrac{1}{\lambda_{min}} = R(1) = R$; $n = \infty$ to1

$$\frac{1}{\lambda_{min}} = R\left\{1 - \frac{1}{4}\right\} = \frac{3R}{4} \text{ ; } n = 2 \text{ to } 1$$

$$\Rightarrow \lambda_{max} - \lambda_{min} = \frac{4}{3R} - \frac{1}{R}$$

$$304 = \frac{1}{3R} \qquad \ldots(a)$$

Paschen : $Y_{\lambda'_{min}} = R\left(\dfrac{1}{9}\right)$ and $Y_{\lambda'_{max}} = R\left(\dfrac{1}{9} - \dfrac{1}{16}\right) = \dfrac{7R}{16 \times 9}$

$$\lambda'_{max} - \lambda'_{min} = \frac{16 \times 9}{7R} - \frac{9}{R} = \frac{81}{7R} \qquad \text{....(b)}$$

$$\frac{(b)}{(a)} = \frac{x}{3 \times 304} = \frac{81}{7} \quad \Rightarrow \quad x = 10553.14$$

{It is not mentioned that the answer should be an integer so better answer should be 10553.14}

PART-B : CHEMISTRY

SECTION – 1 : (Maximum Marks : 80)

Single Choice Type

This section contains 20 Single choice questions. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which Only One is correct.

Full Marks : +4 If ONLY the correct option is chosen.

Negative Marks : –1 (minus one) mark will be deducted for indicating incorrect response.

26. The elements with atomic numbers 101 and 104 belong to, respectively :

(1) Actinoids and Group 4
(2) Group 11 and Group 4
(3) Group 6 and Actinoids
(4) Actinoids and Group 6

Ans. (1)

Sol. $_{89}Ac \longrightarrow {}_{103}Lr$

Belongs to actinoids series and they all belongs to 3rd group. So atomic no. 101 element is actinoids and atomic number 104 element belongs to 4th group.

27. The ionic radii of O^{2-}, F^{-}, Na^{+} and Mg^{2+} are in the order :

(1) $O^{2-} > F^{-} > Na^{+} > Mg^{2+}$
(2) $F^{-} > O^{2-} > Na^{+} > Mg^{2+}$
(3) $O^{2-} > F^{-} > Mg^{2+} > Na^{+}$
(4) $Mg^{2+} > Na^{+} > F^{-} > O^{2-}$

Ans. (1)

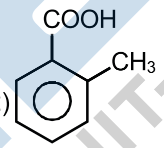
Sol.

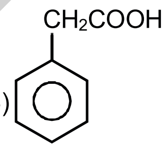
	O^{2-}	F^{-}	Na^{+}	Mg^{2+}
Z	8	9	11	12
No. of e^{-}	10	10	10	10

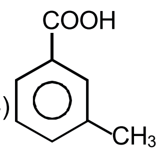
In isoelectronic species greater is Z smaller is radius so order is $O^{2-} > F^{-} > Na^{+} > Mg^{2+}$.

28. [P] on treatment with $Br_2/FeBr_3$ in CCl_4 produced a single isomer $C_8H_7O_2Br$ wile heating [P] with sodalime gas toluene. The compound [P] is :

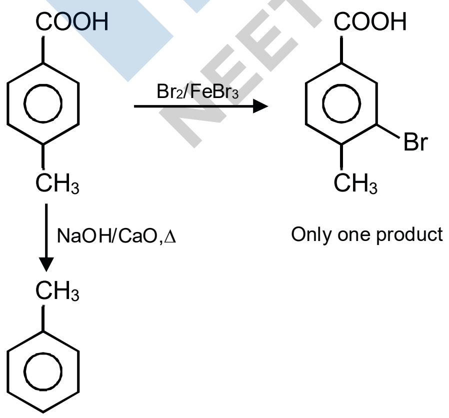
(1) 

(2) 

(3) 

(4) 

Ans. (1)

Sol. 

29. For one mol of an ideal gas, which of these statements must be true ?

(a) U and H each depends only on temperature

(b) Compressibility factor z is not equal to 1

(c) $C_{P,m} - C_{V,m} = R$

(d) $dU = C_V dT$ for any process

(1) (b), (c) and (d) (2) (a) and (c) (3) (a), (c) and (d) (4) (c) and (d)

Ans. (3)

Sol. (a) For ideal gas U and H are function of Temperature $U = \frac{f}{2}nRT$

(c) $C_P - C_V = R$

(d) $\Delta U = C_V dT$ for all processes

30. Among statements (a) – (d), the correct ones are :

(a) Lime stone is decomposed to CaO during the extraction of iron from its oxides.

(b) In the extraction of silver, silver is extracted as an anionic complex.

(c) Nickel is purified by Mond's process.

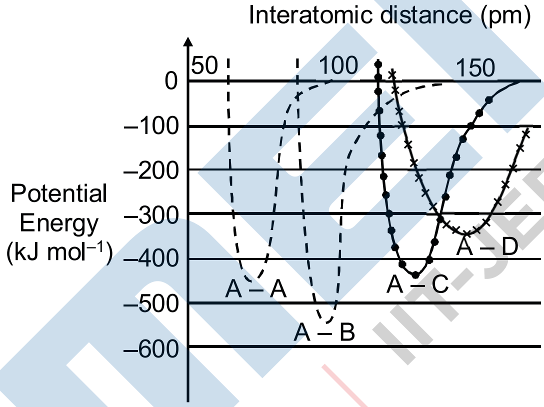
(d) Zr and Ti are purified by Van Arkel method.

(1) (c) and (d) only (2) (a), (b), (c) and (d) (3) (b), (c) and (d) only (4) (a), (c) and (d) only

Ans. (2)

Sol. All statements are correct.

31. The intermolecular potential energy for the molecules A, B, C and D given below suggests that :



(1) A-D has the shortest bond length

(2) A-A has the largest bond enthalpy

(3) D is more electronegative than other atoms.

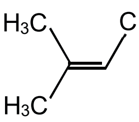
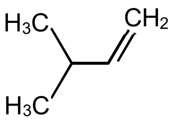
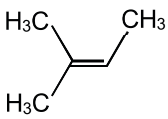
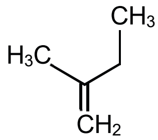
(4) A-B has the stiffest bond.

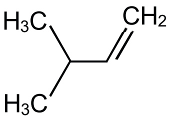
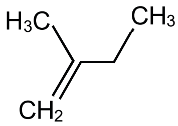
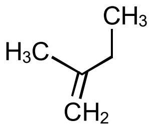
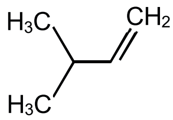
Ans. (4)

Sol. Bond enthalpy of AB bond is highest so A-B bond is more strong and B is highest electronegative atom.

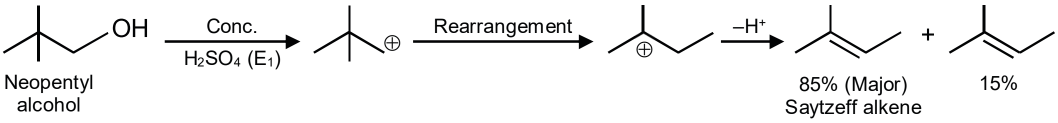
Order of bond length $\Rightarrow$ A–A < A–B < A–C < A–D

32. When neopentyl alcohol is heated with an acid, it shlowly converted into an 85 : 15 mixture of alkenes A and B, respectively. What are these alkenes ?

(1)  and  (2)  and 

(3)  and  (4)  and 

Ans. (2)

Sol. 

33. The region in the electromagnetic spectrum where the Balmer series lines appear is :

(1) Infrared (2) Microwave (3) Ultraviolet (4) Visible

Ans. (4)

Sol. In hydrogen spectrum maximum lines of Balmer series lies in visible region.

34. On combustion of Li, Na and K in excess of air, the major oxides formed, respectively, are :

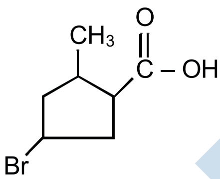
(1) Li_2O, Na_2O and K_2O_2 (2) Li_2O_2, Na_2O_2 and K_2O_2

(3) Li_2O, Na_2O_2 and K_2O (4) Li_2O, Na_2O_2 and KO_2

Ans. (4)

Sol. On heating in excess air Li form oxide sodium form peroxide while K, Rb, Cs form superoxide.

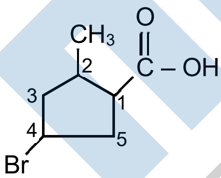
35. The IUPAC name of the following compound is :



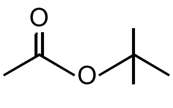
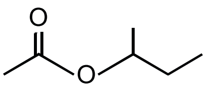
(1) 3-Bromo-5-methylcyclopentane (2) 3-Bromo-5-methylcyclopentanoic

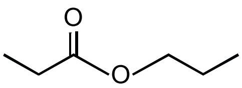
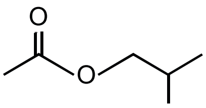
(3) 5-Bromo-5-methylcyclopentanoic (4) 4-Bromo-2-methylcyclopentane carboxylic acid

Ans. (4)

Sol.  4-Bromo-2-methylcyclopentane carboxylic acid

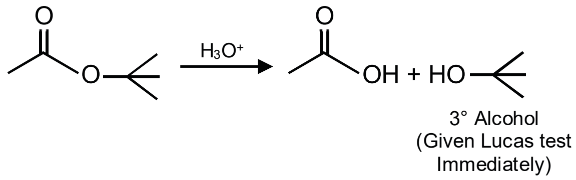
36. An organic compound (A) (molecular formula $C_6H_{12}O_2$) was hydrolysed with dil. H_2SO_4 to give a carboxylic acid (B) and an alcohol (C). 'C' gives white turbidity immediately when treated with anhydrous $ZnCl_2$ and conc. HCl. The organic compound (A) is :

(1)  (2) 

(3)  (4) 

Ans. (1)

Sol.



37. On heating, lead (II) nitrate gives a brown gas (A). The gas (A) on cooling changes to a colourless solid/liquid (B). (B) on heating with NO changes to blue solid (C). The oxidation number of nitrogen in solid (C) is :

(1) +5 (2) +3 (3) +4 (4) +2

Ans. (2)

Sol. $Pb(NO_3)_2 \xrightarrow{\Delta} PbO + 2NO_2(g)$ (brown gas)

'B'

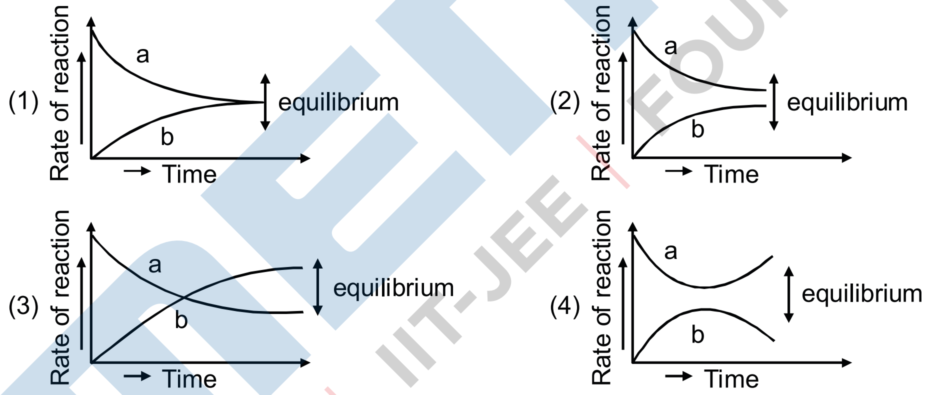
$2NO_2(g) \xrightarrow{\text{cooling}} N_2O_4$

'B' 'C'

$NO_2(g) + NO(g) \longrightarrow \overset{+3}{N_2}O_3$

'D'

38. For the equilibrium $A \rightleftharpoons B$, the variation of the rate of the forward (a) and reverse (b) reaction with time is given by :



Ans. (1)

Sol. At equilibrium, rate of forward reaction = Rate of backward reaction.

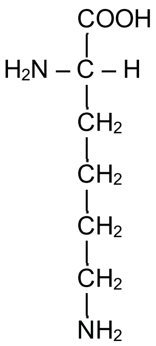
39. Which of the following will react with $CHCl_3$ + alc. KOH ?

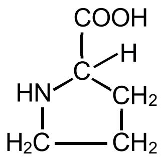
(1) Adenine and lysine (2) Adenine and thymine

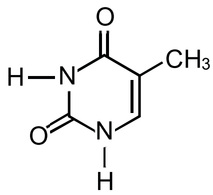
(3) Adenine and proline (4) Thymine and proline

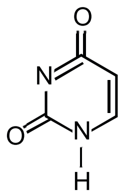
Ans. (1)

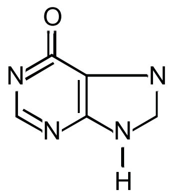
Sol. Compounds having 1° amine give carbylamine reaction with $CHCl_3$ & alc. KOH


* Lysine


Proline


Thymine (T)


Cytosine (C)

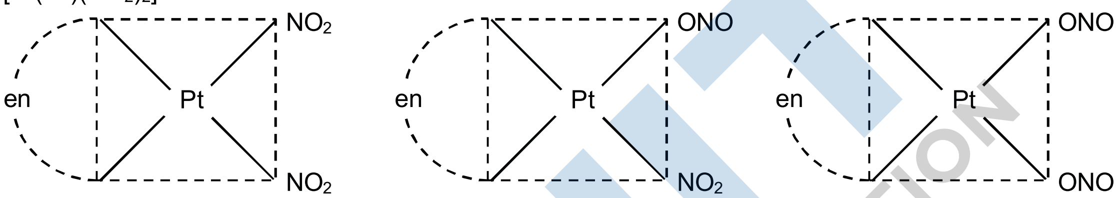

Adenine (A)

40. The number of isomers possible for $[Pt(en)(NO_2)_2]$ is :

(1) 1 (2) 2 (3) 3 (4) 4

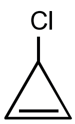
Ans. (3)

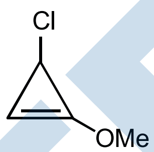
Sol. $[Pt(en)(NO_2)_2]$



So, total possible isomers are 3.

41. The decreasing order of reactivity of the following organic molecules towards $AgNO_3$ solution is :


(A)


(B)

$CH_3\underset{\substack{|\\ Cl}}{C}HCH_3$
(C)

$CH_3\underset{\substack{|\\ Cl}}{C}HCH_2NO_2$
(D)

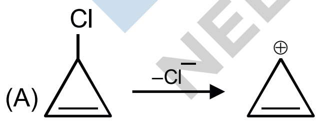
(1) (C) > (D) > (A) > (B) (2) (A) > (B) > (C) > (D)

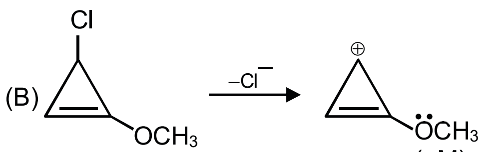
(3) (B) > (A) > (C) > (D) (4) (A) > (B) > (D) > (C)

Ans. (3)

Sol. Given reaction is S_N1 reaction. In S_N1 reaction

$\boxed{\text{Rate of reaction} \propto \text{Stability of } C^+}$


Aromatic
(i)


(+M)
Aromatic & +M
(ii)

(C) $CH_3-\underset{\substack{|\\ Cl}}{C}H-CH_3 \xrightarrow{-Cl^-} CH_3-\overset{\oplus}{C}H-CH_3$
(2°)
(iii)

(D) $CH_3-\underset{\substack{|\\ Cl}}{C}H-CH_2-NO_2 \xrightarrow{-Cl^-} CH_3-\overset{\oplus}{C}H-CH_2-NO_2$
(iv) (–M)

Stability of C^+ : ii > i > iii > iv

Reactivity order : B > A > C > D

42. Match the following :

(i) Foam	(a) smoke
(ii) Gel	(b) cell fluid
(iii) Aerosol	(c) jellies
(iv) Emulsion	(d) rubber
	(e) froth
	(f) milk

(1) (i) – (d), (ii) – (b), (iii) – (a), (iv) – (e)
(2) (i) – (e), (ii) – (c), (iii) – (a), (iv) – (f)
(3) (i) – (d), (ii) – (b), (iii) – (e), (iv) – (f)
(4) (i) – (b), (ii) – (c), (iii) – (e), (iv) – (d)

Ans. (2)

Sol. From NCERT [Surface chemistry]

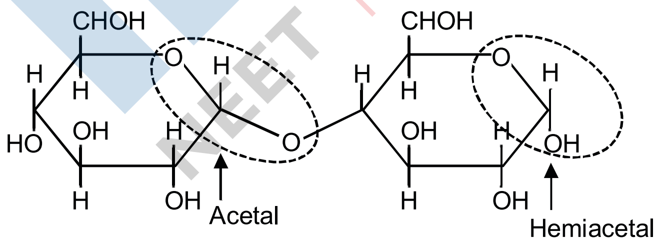
Dispersed phase	Dispersion phase	Type of colloid	Examples
Solid	Solid	Solid sol	Some coloured glasses and gem stones
Solid	Liquid	Sol	Paints, cell fluids
Solid	Gas	Aerosol	Smoke, dust
Liquid	Solid	Gel	Cheese, butter, jellies
Liquid	Liquid	Emulsion	Milk, hair cream
Liquid	Gas	Aerosol	Fog, mist, cloud, insecticide sprays
Gas	Solid	Solid sol	Pumice stone, foam rubber
Gas	Liquid	Foam	Froth, whipped cream, soap lather

43. What are the functional groups present in the structure of maltose ?

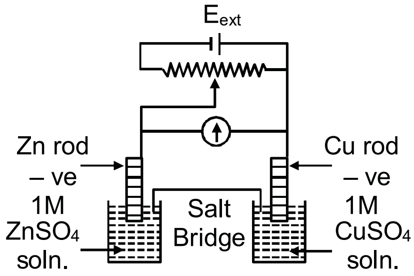
(1) Two acetals
(2) One acetal and one ketal
(3) One ketal and one hemiketal
(4) One acetal and one hemiacetal

Ans. (4)

Sol.



Maltose

44. 

$E^0_{Cu^{2+}|Cu} = +0.34$ V

$E^0_{Zn^{2+}|Zn} = -0.76$ V

Identify the incorrect statement from the options below for the above cell :

(1) If E_{ext} = 1.1 V, no flow of e^- or current occurs

(2) If E_{ext} < 1.1 V, Zn dissolves at anode and Cu deposits at cathode

(3) If E_{ext} > 1.1 V, e^- flow from Cu to Zn

(4) If E_{ext} > 1.1 V, Zn dissolves at Zn electrode and Cu deposits at Cu electrode

Ans. (4)

Sol. $E^0_{cell} = E^0_{Cu^{2+}|Cu} - E^0_{Zn^{2+}|Zn} = 1.1$ V

so If E = 1.1 V no electron will flow

At E > 1.1 V cell act as electrolytic cell and electron will flow from Cu to Zn.

At E < 1.1 V cell act as electrochemical cell so Zn dissolve and Cu deposit.

45. The pair in which both the species have the same magnetic moment (spin only) is :

(1) $[Cr(H_2O)_6]^{2+}$ and $[Fe(H_2O)_6]^{2+}$

(2) $[Co(OH)_4]^{2-}$ and $[Fe(NH_3)_6]^{2+}$

(3) $[Cr(H_2O)_6]^{2+}$ and $[CoCl_4]^{2-}$

(4) $[Mn(H_2O)_6]^{2+}$ and $[Cr(H_2O)_6]^{2+}$

Ans. (1)

Sol.

	No. of unpaired e^-
$[Cr(H_2O)_6]^{2+} \Rightarrow Cr^{2+} \Rightarrow 3d^4 \Rightarrow t_{2g}^{1,1,1}, eg^{1,0}$	4
$[Fe(H_2O)_6]^{2+} \Rightarrow Fe^{2+} \Rightarrow 3d^6 \Rightarrow t_{2g}^{2,1,1}, eg^{1,1}$	4
$[Fe(NH_3)_6]^{2+} \Rightarrow Fe^{2+} \Rightarrow 3d^6 \Rightarrow t_{2g}^{2,1,1}, eg^{1,1}$	4
$[Co(OH)_4]^{2-} \Rightarrow Co^{2+} \Rightarrow 3d^7 \Rightarrow e^{2,2}, t_2^{1,1,1}$	3
$[CoCl_4]^{2-} \Rightarrow Co^{2+} \Rightarrow 3d^7 \Rightarrow e^{2,2}, t_2^{1,1,1}$	3
$[Mn(H_2O)_6]^{2+}, \Rightarrow Mn^{2+} \Rightarrow 3d^5 \Rightarrow t_{2g}^{1,1,1}, eg^{1,1}$	5

So $[Cr(H_2O)_6]^{2+}$ and $[Fe(H_2O)_6]^{2+}$ have same magnetic moment (spin only)

SECTION – 2 : (Maximum Marks : 20)

This section contains FIVE (05) questions. The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto one digit.

If the numerical value has more than two decimal places truncate/round-off the value upto TWO decimal places.

Full Marks : +4 If ONLY the correct option is chosen.

Zero Marks : 0 In all other cases

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46. A 20.0 mL solution containing 0.2 g impure H_2O_2 reacts completely with 0.316 g of $KMnO_4$ in acid solution. The purity of H_2O_2 (in %) is ………… (mol. wt. of H_2O_2 = 34; mol. wt. of $KMnO_4$ = 158)

Ans. (85)

Sol. Let mass of pure H_2O_2 is x gram

$$\overset{+7}{MnO_4^-} + \overset{-1}{H_2O_2} \longrightarrow \overset{+2}{Mn^{2+}} + \overset{0}{O_2}$$

vf = 5   vf = 2

Eq. of H_2O_2 = Eq. of MnO_4^-

$$\left[\frac{x}{34}\right]2 = \left[\frac{0.316}{158}\right]5$$

x = 0.17

So, % purity of H_2O_2 solution $= \frac{0.17}{0.2} \times 100 = 85\%$

47. The mass of ammonia in grams produced when 2.8 kg of dinitrogen quantitatively reacts with 1 kg of dihydrogen is ………………

Ans. (3400)

Sol. Mol of $N_2 = \frac{2800}{28} = 100$ & Mol of $H_2 = \frac{1000}{2} = 500$

	$N_2(g)$	+ $3H_2(g)$	$\longrightarrow$ $2NH_3(g)$
Initial mol	100	500	Limiting reagent is N_2
Final mol	0	500 – 300	200

Mass of NH_3 formed = 200 × 17 = 3400 gram

48. If 75% of a first order reaction was completed in 90 minutes, 60% of the same reaction would be completed in approximately (in minutes) …………..

(Take : log 2 = 0.30 ; log 2.5 = 0.40)

Ans. (60)

Sol. $$T = \frac{2.303}{K}\log\left[\frac{100}{100 - x\%}\right]$$

$$T_{75\%} = \frac{2.303}{K}\log\left[\frac{100}{25}\right] = 90$$

$$T_{60\%} = \frac{2.303}{K}\log\left[\frac{100}{40}\right]$$

$$\frac{T_{75\%}}{T_{60\%}} = \frac{2\log 2}{\log 2.5} \Rightarrow \frac{90}{T_{60\%}} = \frac{2\times 0.3}{0.4}$$

$$T_{60\%} = \frac{90\times 4}{6} = 60 \text{ min.}$$

49. At 300 K, the vapour pressure of a solution containing 1 mol of n-hexane and 3 moles of n-heptane is 550 mm of Hg. At the same temperature, if one more mol of n-heptane is added to this solution, the vapour pressure of the solution increases by 10 mm of Hg. What is the vapour pressure in mmHg of heptane in its pure state .... ?

Ans. (600)

Sol. $P_{total} = P^o_{hexane} \cdot X_{hexane} + P^o_{heptane} \cdot X_{heptane}$

$$550 = [P^o_{hexane}]\times\frac{1}{4} + [P^o_{heptane}]\times\frac{3}{4} \quad \text{....(i)}$$

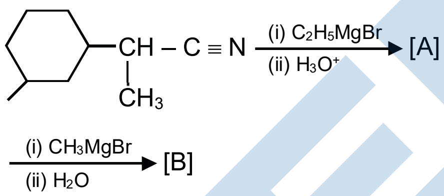
After mixing 1 mol n-heptane

$$560 = [P^o_{hexane}]\times\frac{1}{5} + [P^o_{heptane}]\times\frac{4}{5} \quad \text{....(ii)}$$

On solving eq. (i) and (ii)

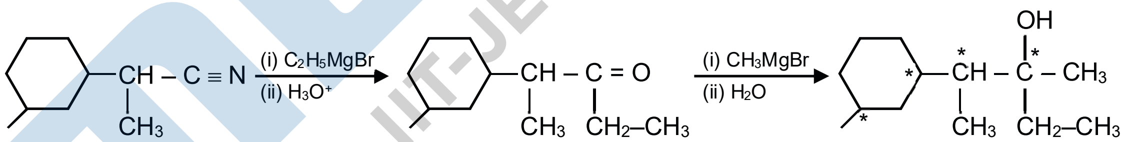
$P^o_{heptane}$ = 600 mm of Hg

50. The number of chiral centres present in [B] is ....



Ans. (4)

Sol.



PART-C : MATHEMATICS

SECTION – 1 : (Maximum Marks : 80)

Single Choice Type

This section contains 20 Single choice questions. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which Only One is correct.

Full Marks : +4 If ONLY the correct option is chosen.

Negative Marks : –1 (minus one) mark will be deducted for indicating incorrect response.

51. Let y = y(x) be the solution of the differential equation, $xy' - y = x^2 (x\cos x + \sin x)$, x > 0. If $y(\pi) = \pi$, then $y''\left(\frac{\pi}{2}\right) + y\left(\frac{\pi}{2}\right)$ is equal to :

(1) $1+\frac{\pi}{2}$ (2) $1+\frac{\pi}{2}+\frac{\pi^2}{4}$ (3) $2+\frac{\pi}{2}+\frac{\pi^2}{4}$ (4) $2+\frac{\pi}{2}$

Ans. (4)

Sol. Given $x\frac{dy}{dx} - y = x^2(x\cos x + \sin x)$

$\Rightarrow \frac{dy}{dx} - \frac{1}{x}y = x(x\cos x + \sin x)$ $\therefore$ $I.F. = e^{\int -\frac{1}{x}dx} = e^{-\ell nx} = \frac{1}{x}$

$\therefore$ solution is $y.\frac{1}{x} = \int \frac{1}{x}\cdot x(x\cos x + \sin x)dx + C$

$\frac{y}{x} = \int (x\cos x + \sin x)dx + C$

$\frac{y}{x} = x\sin x + C, \frac{\pi}{\pi} = 0 + C \Rightarrow C = 1$

$y = x^2 \sin x + x$

$\frac{dy}{dx} = x^2\cos x + 2x\sin x + 1$

$\frac{d^2y}{dx^2} = -x^2 \sin x + 2x\cos x + 2x\cos x + 2\sin x$ $= -x^2 \sin x + 4x\cos x - 2\sin x$

$\therefore y''\left(\frac{\pi}{2}\right) + y\left(\frac{\pi}{2}\right) = \left(-\frac{\pi}{4} + 4.0 + 2\right) + \left(\frac{\pi^2}{4}\cdot 1 + \frac{\pi}{2}\right)$

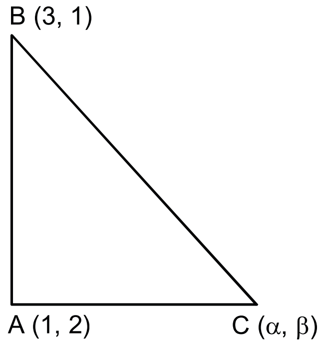
$= -\frac{\pi^2}{4} + 2 + \frac{\pi^2}{4} + \frac{\pi}{2}$

$= \frac{\pi}{2} + 2$

52. A triangle ABC lying in the first quadrant has two vertices as A(1, 2) and B(3, 1). If $\angle BAC = 90°$, and $ar(\Delta ABC) = 5\sqrt{5}$ sq. units, then the abscissa of the vertex C is :

(1) $1+\sqrt{5}$ (2) $2+\sqrt{5}$ (3) $1+2\sqrt{5}$ (4) $2\sqrt{5}-1$

Ans. (3)



Sol. $m_{AC} = \dfrac{\beta-2}{\alpha-1}$

$m_{AB} = \dfrac{2-1}{1-3} = -\dfrac{1}{2}$

$AB \perp AC \quad \therefore \quad \dfrac{\beta-2}{\alpha-1}\left(-\dfrac{1}{2}\right) = -1$

$\beta = 2\alpha - 2 + 2 \Rightarrow \beta = 2\alpha$

Now area of $\Delta ABC = 5\sqrt{5} = \dfrac{1}{2} AB \cdot AC$

$\Rightarrow \quad \dfrac{1}{2}\sqrt{(3-1)^2 + (1-2)^2} \cdot \sqrt{(\alpha-1)^2 + (\beta-2)^2} = 5\sqrt{5}$

$\Rightarrow \quad \sqrt{(\alpha-1)^2 + (2\alpha-2)^2} = 10$

$\Rightarrow \quad \sqrt{(\alpha-1)^2}\sqrt{5} = 10 \quad \Rightarrow \quad |\alpha-1| = 2\sqrt{5} \quad \Rightarrow \quad \alpha = 1 \pm 2\sqrt{5}$

53. Let f(x) = |x – 2| and g(x) = f(f(x)), $x \in [0, 4]$. Then $\int_0^3 (g(x) - f(x))\,dx$ is equal to :

(1) 0 (2) $\dfrac{1}{2}$ (3) $\dfrac{3}{2}$ (4) 1

Ans. (4)

Sol. $f(x) = |x-2| = \begin{cases} 2-x & x<2 \\ x-2 & x \ge 2 \end{cases}$

$g(x) = f(f(x)) = \begin{cases} 2-f(x) & f(x)<2 \\ f(x)-2 & f(x) \ge 2 \end{cases}$

$= \begin{cases} 2-(2-x) & 2-x<2 & x<2 \\ (2-x)-2 & 2-x \ge 2 & x<2 \\ 2-(x-2) & x-2<2 & x \ge 2 \\ (x-2)-2 & x-2 \ge 2 & x \ge 2 \end{cases} = \begin{cases} x & 0<x<2 \\ -x & x \le 0 \\ 4-x & 2 \le x < 4 \\ x-4 & x \ge 4 \end{cases}$

$\int_0^3 (g(x) - f(x))\,dx = \int_0^2 x\,dx + \int_2^3 (4-x)\,dx - \int_0^3 |x-2|\,dx = 1$

54. The integral $\int \left(\dfrac{x}{x\sin x + \cos x}\right)^2 dx$ is equal to (where C is a constant of integration) :

(1) $\tan x + \dfrac{x\sec x}{x\sin x + \cos x} + C$ (2) $\tan x - \dfrac{x\sec x}{x\sin x + \cos x} + C$

(3) $\sec x + \dfrac{x\tan x}{x\sin x + \cos x} + C$ (4) $\sec x - \dfrac{x\tan x}{x\sin x + \cos x} + C$

Ans. (2)

Sol. $\int \dfrac{x^2}{(x\sin x + \cos x)^2} = \int \dfrac{x}{\cos x} \cdot \dfrac{x\cos x}{(x\sin x + \cos x)^2}\,dx$

$$= \frac{x}{\cos x}\int \frac{x\cos x}{(x\sin x+\cos x)^2}dx - \int\left[\frac{d}{dx}(x\sec x)\int \frac{x\cos x}{(x\sin x+\cos x)^2}dx\right]dx$$

$$= \frac{x}{\cos x}\left(-\frac{1}{x\sin x+\cos x}\right)+\int \sec^2 x\,dx$$

$$\left(\because \quad \int \frac{x\cos x}{(x\sin x+\cos x)^2}dx = \frac{-1}{x\sin x+\cos x} \quad \text{and}\right.$$

$$\left.\frac{d}{dx}(x\sec x) = \sec x + x\sec x\tan x = \sec x\left(1+\frac{x\sin x}{\cos x}\right) = \sec^2 x(x\sin x+\cos x)\right)$$

$$= \frac{-x}{\cos x(x\sin x+\cos x)}+\frac{\sin x}{\cos x}+C = \tan x - \frac{x\sec x}{x\sin x+\cos x}+C$$

55. If $1 + (1 - 2^2.1) + (1 - 4^2.3) + (1 - 6^2.5) + \ldots\ldots + (1 - 20^2.19) = \alpha - 220\beta$ then an ordered pair (α, β) is equal to :

(1) (11, 97) (2) (10, 103) (3) (11, 103) (4) (10, 97)

Ans. (3)

Sol. $$S = 1+\sum_{r=1}^{10} 1-(2r)^2(2r-1) = 1+10-\sum_{r=1}^{10}(8r^3-4r^2) = 11-\left[8\left(\frac{10\times 11}{2}\right)^2-4\left(\frac{10\times 11\times 21}{6}\right)\right]$$

$= 11 - [2(110)^2 - 140 \times 11]$

$= 11 - 22(1100 - 70)$

$= 11 - 220(110 - 7)$

$\therefore \quad \alpha - 220\beta$

$= 11 - 220(103)$

$\therefore \quad \alpha = 11, \beta = 103$

$(\alpha, \beta) = (11, 103)$

56. A survey shows that 63% of the people in a city read newspaper A whereas 76% read news paper B. If x% of the people read both the newspapers, then a possible value of x can be :

(1) 65 (2) 55 (3) 37 (4) 29

Ans. (2)

Sol. $n(A) = 63\%$

$n(B) = 76\%$

$n(A \cap B) = x\%$

Let $n(\cup) = 100$

$n(A) = 63, n(B) = 76, n(A \cap B) = x$

$n(A \cup B) = n(A) + n(B) - n(A \cap B) \le 100$

$63 + 76 - x \le 100$

$x \ge 39$

but $n(A \cap B) \le n(A) \quad \therefore \quad 39 \le x \le 63$

57. Two vertical poles AB = 15 m and CD = 10 m are standing apart on a horizontal ground with points A and C on the ground. If P is the point of intersection of BC and AD, then the height of P (in m) above the line AC is :

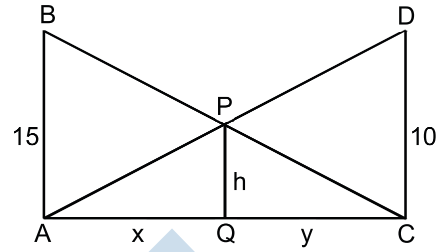
(1) 10/3 (2) 6 (3) 5 (4) 20/3

Ans. (2)

Sol. $\Delta AQP \sim \Delta ACD \Rightarrow \frac{x}{h} = \frac{x+y}{10}$ .....(1)

$\because$ $\Delta CQP \sim \Delta CAB \Rightarrow \frac{y}{h} = \frac{x+y}{15}$ .....(2)

(1) + (2) $\rightarrow \frac{x+y}{h} = (x+y)\left(\frac{1}{10}+\frac{1}{15}\right) \Rightarrow h = 6$



58. Let P(3, 3) be a point on the hyperbola, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal to it at P intersects the x-axis at (9,0) and e is its eccentricity, then the ordered pair (a^2, e^2) is equal to :

(1) $\left(\frac{9}{2},3\right)$ (2) $\left(\frac{3}{2},2\right)$ (3) (9,3) (4) $\left(\frac{9}{2},2\right)$

Ans. (1)

Sol. Hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

P(3,3) lies on hyperbola then $\frac{1}{a^2} - \frac{1}{b^2} = \frac{1}{9}$ ....(1)

Normal at (3,3) is

$\frac{a^2x}{3} + \frac{b^2y}{3} = a^2 + b^2$

pass through (9,0)

$3a^2 = a^2 + b^2 = 2a^2 = b^2$

then $\frac{1}{a^2} - \frac{1}{2a^2} = \frac{1}{9}$

$2a^2 = 9 \Rightarrow a^2 = \frac{9}{2}$ and $b^2 = 9$

$e^2 = 1 + \frac{b^2}{a^2} = 1 + 2 = 3$

59. Let [t] denote the greatest integer $\leq$ t. Then the equation in x, $[x]^2 + 2[x + 2] - 7 = 0$ has :

(1) no integral solution (2) infinitely many solutions

(3) exactly two solutions (4) exactly four integral solutions

Ans. (2)

Sol. $[x]^2 + 2[x + 2] - 7 = 0$

$[x]^2 + 2([x] + 2) - 7 = 0$

Let [x] = t

$t^2 + 2t - 3 = 0$

$t = 1, -3$

$[x] = -3, 1$

$x \in [-3, -2) \cup [1, 2)$

Hence equation has infinitely many solutions.

60. If $(a+\sqrt{2}\,b\cos x)(a-\sqrt{2}\,b\cos y) = a^2 - b^2$, where a > b > 0, then $\frac{dx}{dy}$ at $\left(\frac{\pi}{4},\frac{\pi}{4}\right)$ is :

(1) $\frac{2a+b}{2a-b}$ (2) $\frac{a+b}{a-b}$ (3) $\frac{a-b}{a+b}$ (4) $\frac{a-2b}{a+2b}$

Ans. (2)

Sol. $(a+\sqrt{2}\,b\cos x)(a-\sqrt{2}\,b\cos y) = a^2 - b^2$

$$(a-\sqrt{2}\,b\cos y)(-\sqrt{2}\,b\sin x)\frac{dx}{dy} + \sqrt{2}\,b\sin y(a+\sqrt{2}\,b\cos x) = 0$$

$$\frac{dx}{dy} = \frac{\sqrt{2}b\sin y(a+\sqrt{2}b\cos x)}{\sqrt{2}b\sin x(a-\sqrt{2}b\cos y)} = \frac{\sin y(a+\sqrt{2}b\cos x)}{\sin x(a-\sqrt{2}b\cos y)}$$

$$\left.\frac{dx}{dy}\right|_{\left(\frac{\pi}{4},\frac{\pi}{4}\right)} = \frac{a+b}{a-b}$$

61. Let x_0 be the point of local maxima of $= \vec{a}.(\vec{b}\times\vec{c})$, where $\vec{a} = x\hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = -2\hat{i} + x\hat{j} - \hat{k}$ and $\vec{c} = 7\hat{i} - 2\hat{j} + x\hat{k}$. Then the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$ at $x = x_0$ is :

(1) –22 (2) 14 (3) –4 (4) –30

Ans. (1)

Sol. $f(x) = \left[\vec{a}\;\vec{b}\;\vec{c}\right] = \begin{vmatrix} x & -2 & 3 \\ -2 & x & -1 \\ 7 & -2 & x \end{vmatrix} = x(x^2 - 2) + 2(-2x + 7) + 3(4 - 7x) = x^3 - 2x - 4x + 14 + 12 - 21x$

$f(x) = x^3 - 27x + 26$

+ – +

–3 3

$f'(x) = 3x^2 - 27 = 3(x - 3)(x + 3)$

so local maxima point is $x_0 = -3$

Now $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = -2x - 2x - 3 - 14 - 2x - x + 7x + 4 + 3x = 3x - 13$

at $x = x_0 = -3$

$\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = -9 - 13 = -22$

62. Let f(x) $\int \frac{\sqrt{x}}{(1+x)^2}dx$ $(x \geq 0)$. Then f(3) – f(1) is equal to :

(1) $-\frac{\pi}{6}+\frac{1}{2}+\frac{\sqrt{3}}{4}$ (2) $-\frac{\pi}{12}+\frac{1}{2}+\frac{\sqrt{3}}{4}$ (3) $\frac{\pi}{12}+\frac{1}{2}-\frac{\sqrt{3}}{4}$ (4) $\frac{\pi}{6}+\frac{1}{2}-\frac{\sqrt{3}}{4}$

Ans. (3)

Sol. $f(x) = \int \frac{\sqrt{x}}{(1+x)^2}dx$

Let $x = \tan^2\theta$

$dx = 2\tan\theta \sec^2\theta\, d\theta$

$$f(x) = \int \frac{\tan\theta}{(1+\tan^2\theta)^2} . 2\tan\theta \sec^2\theta d\theta$$

$$f(x) = \int \frac{\tan\theta}{\sec^4\theta} . 2\tan\theta \sec^2\theta d\theta$$

$$f(x) = \int 2\tan^2\theta \cos^2\theta d\theta$$

$$f(x) = \int 2\sin^2\theta d\theta$$

$$f(x) = \int (1-\cos 2\theta) d\theta$$

$$f(x) = \theta - \frac{\sin 2\theta}{2} + C = \theta - \frac{\tan\theta}{1+\tan^2\theta} + C$$

$$f(x) = \tan^{-1}\sqrt{x} - \frac{\sqrt{x}}{1+x} + C$$

now $f(3) - f(1) = \tan^{-1}\sqrt{3} - \frac{\sqrt{3}}{1+3} - \tan^{-1}\sqrt{1} + \frac{1}{1+1}$

$$= \frac{\pi}{3} - \frac{\pi}{4} + \frac{1}{2} - \frac{\sqrt{3}}{4} = \frac{\pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{4}$$

63. The mean and variance of 8 observations are 10 and 13.5, respectively. If 6 of these observations are 5, 7, 10, 12, 14, 15, then the absolute difference of the remaining two observations is :

(1) 9 (2) 5 (3) 3 (4) 7

Ans. (4)

Sol. Let remaining two observations are a and b

$\frac{5+7+12+10+15+14+a+b}{8} = 10 \quad \Rightarrow 63 + a + b = 80 \Rightarrow a + b = 17$ ....(1)

$$\sigma^2 = \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2$$

$$\Rightarrow 13.5 = \frac{25+49+144+100+225+196+a^2+b^2}{8} - 100$$

$908 = a^2 + b^2 + 739$

$a^2 + b^2 = 169$

$(a + b)^2 - 2ab = 169$

$289 - 169 = 2ab \Rightarrow ab = 60$

$\therefore |a - b|^2 = (a + b)^2 - 4ab = 289 - 240 = 49$

$\therefore |a - b| = 7$

64. Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(a > b)$ be a given ellipse, length of whose latus rectum is 10. If its eccentricity is the maximum value of the function $\phi(t) = \frac{5}{12} + t - t^2$ then $a^2 + b^2$ is equal to :

(1) 145 (2) 126 (3) 116 (4) 135

Ans. (2)

Sol. L.R. $= \frac{2b^2}{a} = 10 \quad \Rightarrow \quad b^2 = 5a$

$\phi(t) = \frac{5}{12} - \left(t^2 - t + \frac{1}{4} - \frac{1}{4}\right) = \frac{5}{12} + \frac{1}{4}\left(t - \frac{1}{2}\right)^2$

$= \frac{2}{3} - \left(t - \frac{1}{2}\right)^2$

max $\phi(t) = \frac{2}{3} = e$

$b^2 = a^2 (1 - e^2)$

$5a = a^2\left(1 - \frac{4}{9}\right) \Rightarrow \quad 5 = \frac{5}{9}a \Rightarrow \quad a^2 = 81,\ b^2 = 45$

$a^2 + b^2 = 126$

65. Let f be a twice differentiable function on (1, 6). If f(2) = 8, f'(2) = 5, $f'(x) \geq 1$ and $f''(x) \geq 4$, for all $x \in (1,6)$, then :

(1) $f(5) + f'(5) \geq 28$ (2) $f'(5) + f''(5) \leq 20$ (3) $f(5) \leq 10$ (4) $f(5) + f'(5) \leq 26$

Ans. (1)

Sol. Given $f'(x) \geq 1 \Rightarrow \int_2^5 f'(x)dx \geq \int_2^5 1dx$

$\Rightarrow (f(x))_2^5 \geq (x)_2^5 \quad \Rightarrow f(5) - f(2) \geq 3 \quad \Rightarrow f(5) \geq 11$ ......(1)

Now $f''(x) \geq 4 \Rightarrow \int_2^5 f''(x)dx \geq \int_2^5 4dx$

$\Rightarrow (f'(x))_2^5 \geq (4x)_2^5$

$\Rightarrow f'(5) - f'(2) \geq 12$

$\Rightarrow f'(5) \geq 17$ .... (2)

(1) + (2) $\Rightarrow f(5) + f'(5) \geq 28$

66. Let $u = \frac{2z+i}{z-ki}$, z = x + iy and k > 0. If the curve represented by Re (u) + Im(u) = 1 intersects the y-axis at points P and Q where PQ = 5 then the value of k is

(1) $\frac{3}{2}$ (2) $\frac{1}{2}$ (3) 4 (4) 2

Ans. (4)

Sol. $u = \frac{2(x+iy)+i}{(x+iy)-ki} = \frac{2x+(2y+1)i}{x+(y-k)i} \times \frac{x-(y-k)i}{x-(y-k)i}$

Real part of u = Re(u) $= \frac{2x^2 + (2y+1)(y-k)}{x^2+(y-k)^2}$

Imaginary part of u = Im(u) $= \frac{x(2y+1) - 2x(y-k)}{x^2+(y-k)^2}$

Now Re(u) + Im(u) = 1

$$\frac{2x^2+(2y+1)(y-k)+x(2y+1)-2x(y-k)}{x^2+(y-k)^2}=1$$

for y-axis put x = 0 $\Rightarrow \frac{(2y+1)(y-k)}{(y-k)^2}=1$

$\Rightarrow$ (2y + 1) (y – k) = (y – k)^2

$\Rightarrow$ (y – k) (y + (1 + k) = 0

y = k, –(1 + k)

Now point P(0, k), Q (0,– (1 + k))

PQ = |2k + 1| = 5

2k + 1 = ± 5

2k = 4, – 6

k = 2 – 3

hence k = 2 (k > 0)

67. If $A=\begin{bmatrix}\cos\theta & i\sin\theta\\ i\sin\theta & \cos\theta\end{bmatrix}, \left(\theta=\frac{\pi}{24}\right)$ and $A^5=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$, where $i=\sqrt{-1}$, then which one of the following is not true?

(1) $0 \le a^2 + b^2 \le 1$ (2) $a^2 - d^2 = 0$ (3) $a^2 - b^2 = \frac{1}{2}$ (4) $a^2 - c^2 = 1$

Ans. (3)

Sol. $A^2=\begin{bmatrix}\cos\theta & i\sin\theta\\ i\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\cos\theta & i\sin\theta\\ i\sin\theta & \cos\theta\end{bmatrix}=\begin{bmatrix}\cos^2\theta-\sin^2\theta & 2i\sin\theta\cos\theta\\ 2i\sin\theta\cos\theta & \cos^2\theta-\sin^2\theta\end{bmatrix}=\begin{bmatrix}\cos2\theta & i\sin2\theta\\ i\sin2\theta & \cos2\theta\end{bmatrix}$

$A^3=\begin{bmatrix}\cos2\theta & i\sin2\theta\\ i\sin2\theta & \cos2\theta\end{bmatrix}\begin{bmatrix}\cos\theta & i\sin\theta\\ i\sin\theta & \cos\theta\end{bmatrix}$

$=\begin{bmatrix}\cos3\theta & i\sin3\theta\\ i\sin3\theta & \cos3\theta\end{bmatrix}$

$A^5=\begin{bmatrix}\cos5\theta & i\sin5\theta\\ i\sin5\theta & \cos5\theta\end{bmatrix}=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$

a = cos5θ, b = isin5θ, c = isin5θ, d = cos5θ

a = d, b = c

(1) $a^2 + b^2 = \cos^2 5\theta - \sin^2 5\theta = \cos10\theta \in (0,1)$ as $\theta = \frac{\pi}{24}$

(2) $a^2 - d^2 = 0$

(3) $a^2 - b^2 = \cos^2 5\theta + \sin^2 5\theta = 1$

(4) $a^2 - c^2 = a^2 - b^2 = 1$

68. Let α and β be the roots of $x^2 - 3x + p = 0$ and γ and δ be the roots of $x^2 - 6x + q = 0$. If $\alpha, \beta, \gamma, \delta$ from a geometric progression. Then ratio (2q + p) : (2q – p) is

(1) 3 : 1 (2) 5 : 3 (3) 9 : 7 (4) 33 : 31

Ans. (3)

Sol. $\alpha = a,\ \beta = ar,\ \gamma = ar^2,\ \delta = ar^3$

$\alpha + \beta = 3 \Rightarrow a + ar = 3$ ....(1)

$\gamma + \delta = 6 \Rightarrow ar^2 + ar^3 = 6$ ....(2)

By (1) and (2) $\dfrac{ar^2(1+r)}{a(1+r)} = \dfrac{6}{3} \Rightarrow r^2 = 2$

$$\therefore \quad \frac{2q+p}{2q-p} = \frac{2\gamma\delta+\alpha\beta}{2\gamma\delta-\alpha\beta} = \frac{2a^2r^5+a^2r}{2a^2r^5-a^2r} = \frac{2r^4+1}{2r^4-1} = \frac{8+1}{8-1} = \frac{9}{7}$$

69. The value of $\sum_{r=0}^{20} {}^{50-r}C_6$ is equal to :

(1) ${}^{51}C_7 - {}^{30}C_7$ (2) ${}^{51}C_7 + {}^{30}C_7$ (3) ${}^{50}C_7 - {}^{30}C_7$ (4) ${}^{50}C_6 - {}^{30}C_6$

Ans. (1)

Sol. $\sum_{r=0}^{20} {}^{50-r}C_6 =$

$= {}^{50}C_6 + {}^{49}C_6 + {}^{48}C_6 + \ldots\ldots + {}^{30}C_6 = -{}^{30}C_7 + {}^{30}C_7 + {}^{30}C_6 + {}^{31}C_6 + {}^{32}C_6 + \ldots\ldots + {}^{50}C_6$

$= -{}^{30}C_7 + {}^{31}C_7 + {}^{31}C_6 + {}^{32}C_6 + \ldots\ldots + {}^{50}C_6$

$= -{}^{30}C_7 + {}^{32}C_7 + {}^{32}C_6 + \ldots\ldots + {}^{50}C_6$

$= -{}^{30}C_7 + {}^{51}C_7 = {}^{51}C_7 - {}^{30}C_7$

70. Given the following two statements :

(S_1) : $(q \vee p) \to (p \leftrightarrow \sim q)$ is a tautology.

(S_2) : $\sim q \wedge (\sim p \leftrightarrow q)$ is a fallacy. Then :

(1) only (S_2) is correct. (2) both (S_1) and (S_2) are not correct.

(3) both (S_1) and (S_2) are correct. (4) only (S_1) is correct.

Ans. (2)

Sol.

p	q	$\sim p$	$\sim q$	$q \vee p$	$p \leftrightarrow \sim q$	$S_1 : (q \vee p) \to (p \leftrightarrow \sim q)$	$\sim p \leftrightarrow q$	$S_2 : \sim q \wedge (\sim p \leftrightarrow q)$
T	T	F	F	T	F	F	F	F
T	F	F	T	T	T	T	T	T
F	T	T	F	T	T	T	T	F
F	F	T	T	F	F	T	F	F

so both (S_1) and (S_2) are not correct.

SECTION – 2 : (Maximum Marks : 20)

This section contains FIVE (05) questions. The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto one digit.

If the numerical value has more than two decimal places truncate/round-off the value upto TWO decimal places.

Full Marks : +4 If ONLY the correct option is chosen.

Zero Marks : 0 In all other cases

71. If the equation of a plane P, passing through the intersection of the planes, $x + 4y - z + 7 = 0$ and $3x + y + 5z = 8$ is $ax + by + 6z = 15$ for some $a, b \in R$, then the distance of the point $(3, 2, -1)$ from the plane P is ……

Ans. 3

Sol. Equation of plane of passing through intersection of planes $x + 4y - z + 7 = 0$ …..(1)

and $3x + y + 5z - 8 = 0$ ………(2)

is $P_1 + \lambda P_2 = 0$

$(x + 4y - z + 7) + \lambda(3x + y + 5z - 8) = 0$

$(3\lambda + 1)x + (\lambda + 4)y + (5\lambda - 1)z + (7 - 8\lambda) = 0$ …..(3)

$ax + by + 6z - 15 = 0$ …..(4)

comparing (3) and (4)

$$\frac{a}{3\lambda+1} = \frac{b}{\lambda+4} = \frac{6}{5\lambda-1} = \frac{-15}{7-8\lambda}$$

$\therefore$ $42 - 48\lambda = -75\lambda + 15$

$27\lambda = -27$

$\lambda = -1$

$\therefore$ $\frac{a}{-2} = \frac{b}{3} = -1 \Rightarrow a = 2, b = -3$

so plane is $2x - 3y + 6z - 15 = 0$…..(5)

distance of plane (5) from point $(3, 2, -1)$ is $p = \left|\frac{6-6-6-15}{\sqrt{4+9+36}}\right| = \left|-\frac{21}{\sqrt{49}}\right| = \left|-\frac{21}{7}\right| = 3$

72. Suppose a differentiable function $f(x)$ satisfies the identity $f(x + y) = f(x) + f(y) + xy^2 + x^2y$, for all real x and y. If $\lim\limits_{x\to 0}\frac{f(x)}{x} = 1$, then $f'(3)$ is equal to :

Ans. 10

Sol. $f(x + y) = f(x) + f(y) + xy^2 + x^2y$

$f'(x + y) = f'(x) + 0 + y^2 + 2xy$

put $y = -x$

$f'(0) = f'(x) + x^2 - 2x^2$

$1 = f'(x) - x^2$

$f'(x) = 1 + x^2$

$f'(3) = 10$

73. The probability of a man hitting a target is $\frac{1}{10}$. The least number of shots required, so that the probability of his hitting the target at least once is greater than $\frac{1}{4}$, is….

Ans. 3

Sol. $p = \frac{1}{10}, q = \frac{9}{10}$

P (not hitting in n trials) $= \left(\frac{9}{10}\right)^n$

$\therefore$ P (at least one hit) $= 1 - \left(\frac{9}{10}\right)^n > \frac{1}{4}$

$\Rightarrow \left(\frac{9}{10}\right)^n < \frac{3}{4}$

$(0.9)^n < 0.75$

$n = 3 \Rightarrow 0.729 < 0.75$ which is true

74. Let $(2x^2 + 3x + 4)^{10} \sum_{r=0}^{20} a_r x^r$. Then $\frac{a_7}{a_{13}}$ is equal to ……

Ans. 8

Sol. General term $= \frac{10!}{r_1!r_2!r_3!}(2x^2)^{r_1} \cdot (3x)^{r_2} (4)^{r_3}$

$$a_7 = \frac{10!.2^3.3.4^6}{3!1!6!} + \frac{10!.2^2.3^3.4^5}{2!3!5!} + \frac{10!.2.3^5.4^4}{1!5!4!} + \frac{10!.3^7.4^3}{7!3!}$$

$$a_{13} = \frac{10!.2^6.3.4^3}{6!1!3!} + \frac{10!.2^5.3^3.4^2}{5!3!2!} + \frac{10!.2^4.3^5.4}{4!5!1!} + \frac{10!.2^2.3^7.4^0}{3!7!}$$

$$\frac{a_7}{a_{13}} = 2^3 \left[\frac{\frac{10!.3.2^{12}}{3!1!6!} + \frac{10!.3^3.2^9}{2!3!5!} + \frac{10!.3^5.2^6}{1!5!4!} + \frac{10!.3^7.2^3}{7!3!}}{\frac{10!.3.2^{12}}{3!1!6!} + \frac{10!.3^3.2^9}{2!3!5!} + \frac{10!.3^5.2^6}{1!5!4!} + \frac{10!.3^7.2^3}{7!3!}}\right] = 2^3 = 8$$

75. If the system of equations

$x - 2y + 3z = 9$

$2x + y + z = b$

$x - 7y + az = 24$, has infinitely many solutions, then $a - b$ is equal to :

Ans. 5

Sol. For infinitely many solutions $D = D_1 = D_2 = D_3 = 0$

$$D = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & 1 \\ 1 & -7 & a \end{vmatrix} = 0 \quad \Rightarrow \quad 1(-2-3) + 7(1-6) + a(1+4) = 0$$

$\Rightarrow \quad -5 - 35 + 5a = 0 \quad \Rightarrow \quad a = 8$

$$D_1 = \begin{vmatrix} 9 & -2 & 3 \\ b & 1 & 1 \\ 24 & -7 & a \end{vmatrix} = 0 \qquad \Rightarrow \qquad \begin{vmatrix} 9 & -2 & 3 \\ b & 1 & 1 \\ 24 & -7 & a \end{vmatrix} = 0$$

$\Rightarrow$ $-b(-16 + 21) + 1(72 - 72) - 1(-63 + 48) = 0$

$\Rightarrow$ $-5b + 15 = 0$

$\Rightarrow$ $b = 3$

similarly $D_2 = D_3 = 0$ for $a = 8$, $b = 3$

$\therefore$ $a - b = 8 - 3 = 5$